Probability - Basic Principles

A. First of all, what is probability?
Probability is the branch of mathematics that attempts to *predict* answers to questions about the likelihood or chances of an event occurring.

An **event** is a collection of outcomes satisfying a particular condition. The probability of an event can range from **0 (impossible) to 1 (certain)** - probabilities less than 0 or greater than 1 have no meaning**.**

The probability of an event, ***E***, is denoted as **P(*E*).** It is a measure of the *likelihood* that the event will occur. Probability can be expressed as a fraction, decimal, or percent – *i.e.* ½ or 0.5 or 50%

There are three basic types of probability:

- **Empirical or Experimental Probability**

Ex. 1: Calculate the *experimental probability* of rolling a "1" on a six-sided die.
(Roll the die 6 times [the number of *trials* in your experiment] and record the outcome for each roll.)

**Experimental probability approaches theoretical probability as the #of trials approaches _____________.

- **Theoretical Probability**

The **theoretical probability** of an event, *E*, is given by: $P(E) = \frac{n(E)}{n(S)}$

where $0 \leq P(E) \leq 1$

Ex. 2: Calculate the *theoretical probability* of rolling a "1" on a six-sided die.

**The sum of the probabilities of all possible events = ______

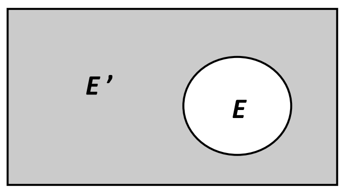
- **Subjective Probability**

Ex. 3: *Estimate* the probability that the next pair of shoes you buy will be the same size as the last pair you bought.   P(*same size*) = _________

B. The Complement of an Event

In probability, the **complement** of an event *E* is written as *E'* (pronounced *E-prime* or *not-E*). *E'* is the event where "event *E* does *not* occur". Thus, whichever outcomes make up event *E*, all the *other* outcomes make up *E'*. Because *E* and *E'* together make up all possible outcomes, *E* + *E'* = 1.

The probability of the **complement** of an event, *E'*, is given by: $\boldsymbol{P(E') = 1 - P(E)}$



Ex. 4: What is the probability that a randomly drawn integer between 1 and 40 is *not* a perfect square?

Ex. 5: Determine the probability of rolling a sum of 7 with a pair of dice.

Ex. 6: Use a tree diagram to illustrate flipping a coin 3 times.
a) Determine the probability of getting HHH ∴ *P*(HHH) = _________
b) Determine the probability of getting at least one T ∴ *P*(at least one T) =________

CW: p. 312-313 #1abcef, 2, 4, 5 (each team has a 50% chance of winning any individual game), 6, 7

Probabilities Using Counting Techniques

In many situations, possible outcomes are not easy or convenient to count individually. In these cases, the counting techniques involving permutations and combinations can be helpful for calculating theoretical probabilities. Yay!

Recall: $P(E) = \dfrac{n(E)}{n(S)}$, $0 \leq P(E) \leq 1$, $1 - P(E) = P(E')$

A. Using Permutations

Recall that when using permutations, **order matters** – whether it is time order or spatial order.

Ex. 1: Alphonse and Beauregard enter a race with 5 friends. The racers draw straws to determine their starting positions. What is the probability that Alphonse will start in lane 1 and Beauregard will start in lane 2?

Ex. 2: A bag contains 26 tiles, each marked with a different letter of the alphabet. What is the probability that a student will take out 4 consecutive letters (without replacement) and have them spell out the word M-A-T-H?

B. Using Combinations

When using combinations, **order is irrelevant** – we are selecting or choosing or “grabbing” some items!

Ex. 3: Suppose you randomly draw 2 marbles, without replacement, from a bag containing six green, four red, and three blue marbles.

a) Determine the probability that both marbles are red.

b) Determine the probability that you pick at least one green marble.

Ex. 4: A focus group of 3 members is to be randomly selected from a medical team consisting of 5 doctors and 7 technicians.

a) What is the probability that the focus group will be comprised entirely of doctors?

b) What is the probability that the focus group will *not* be comprised entirely of doctors?

c) What is the probability that the group will be made up of 1 doctor and 2 technicians?

Ex. 5: In a game of poker, each player is dealt a 5-card hand from a deck of 52 shuffled playing cards.

a) Find the probability of receiving a hand containing a spade flush (all 5 cards are spades).

b) Find the probability of receiving a hand containing a full house of 3 kings and 2 fives.

CW: p. 324-325 #3, 5, 6, 7a, 8, 9, 11

Probability Practise Worksheet

A. Basic Probability Concepts

1. A coin is tossed 3 times.
   a. What is the probability of tossing 3 heads in succession?
   b. What is the probability of tossing exactly 2 heads?
   c. What is the probability of tossing *at least* 2 heads?
2. Two standard dice are rolled.
   a. What is the probability of rolling a pair (both the same number)?
   b. What is the probability of rolling a sum of 9?
   c. What is the probability of rolling a sum that is *less than* 6?
   d. What is the probability that a sum *less than* 7 is *not* rolled?
3. If the probability of rain tomorrow is 65%, what is the probability that it will *not* rain tomorrow?
4. What is the probability of drawing a red face card from a standard deck of playing cards?
5. Use a tree diagram to illustrate the possible outcomes for a couple having 4 children, assuming that the probability of having a boy equals the probability of having a girl. What is the probability that the family will have all 4 boys *or* all 4 girls?
6. A stable has 15 horses available for trail rides. Of these horses, 6 are brown, 5 are mainly white, and the rest are black. If Zeljana selects one at random, what is the probability that her horse will:
   a. be black?
   b. *not* be black?
   c. be either black *or* brown?
7. A number is chosen randomly from the numbers 1 – 20. If event A = {a multiple of 5}, what is the value of $P(A')$?
8. *Match each of these terms with the phrases below.*

a. sample space	b. empirical probability	c. complement	d. subjective probability
e. outcomes	f. theoretical probability	g. trials	h. event

i. deduced from analysis of all possible outcomes _______
ii. the set of all possible outcomes _______
iii. repetitions of a probability experiment _______
iv. the outcomes not included in a particular event _______
v. based upon intuition and previous experience _______
vi. a specific group of outcomes that is being investigated _______
vii. experimental probability _______
viii. the different results of a probability experiment _______

Answers:

1a) 1/8	1b) 3/8	1c) 1/2	2a) 1/6	2b) 1/9	2c) 5/18	2d) 7/12	3) 35%	4) 3/26	5) 1/8
6a) 4/15	6b) 11/15	6c) 2/3	7) 4/5						
8a) ii	8b) vii	8c) iv	8d) v	8e) viii	8f) i	8g) iii	8h) vi		

B. Probabilities Using Counting Techniques

1. A group of volleyball players consists of four grade 11 students and six grade 12 students. If six players are chosen at random to start a match, what is the probability that three will be from each grade?

2. If a bowl contains 10 hazelnuts and 8 almonds, what is the probability that 4 nuts randomly selected from the bowl will all be hazelnuts?

3. Without looking, Adam randomly selects 2 socks from a drawer containing 4 blue, 3 white, and 5 black socks, none of which are paired up. What is the probability that he chooses two socks of the same colour?

4. A four-member curling team is randomly chosen from six grade 10 students and nine grade 11 students. What is the probability that the team has *at least* one grade 10 student?

5. Sam has 5 white and 6 grey huskies in her kennel. If a wilderness expedition chooses a team of 6 sled dogs at random from Sam's kennel, what is the probability that the team will consist of:
   a. all white huskies?
   b. all grey huskies?
   c. 3 of each colour?

6. You need to visit the bank, the book store, the pharmacy, and the video store. You make a random choice of the order in which you visit the four places. Find the probability of each of the following events:
   a. you visit the bank first
   b. you visit the pharmacy second and the book store last
   c. you visit the bank *right* before the pharmacy

Answers:

1) 8/21	2) 7/102	3) 19/66	4) 59/65	5a) 0	5b) 1/462
5c) 100/231	6a) 1/4	6b) 1/12	6c) 1/4		

Independent Events

A. Compound Events

For the past 2 lessons, we have been finding probabilities for *simple events (events that consist of only one outcome).* However, there are times when we may deal with probabilities involving **two or more separate events**. For example, flipping a coin and then rolling a die is an example of two separate events, known as **compound events**.

B. Independent Events

In some situations involving compound events, the occurrence of one event, *A,* has *no effect* on the occurrence of another event, *B*. In such cases, events *A* and *B* are **independent**.

Ex. 1: A coin is tossed 4 times and turns up heads each time. What is the probability that the fifth toss will be heads?

Does the result of the fifth trial depend on the results of the previous 4 trials? No – the coin has no "memory" of the past 4 trials, so each toss is independent!

C. Product Rule for Independent Events

A compound probability asks us to find the likelihood that event A *and* event B will occur. Like the *Fundamental Counting Principle* we used with permutations and combinations, compound probabilities can be found by multiplying the probabilities of each independent event together.

In general, the compound probability of two independent events can be calculated using the product rule for independent events:

P(*A* and *B*) = P(*A*) x P(*B*)

Ex. 2: A coin is flipped while a six-sided die is rolled. What is the probability of flipping heads *and* rolling a 5 in a single trial?

Ex. 3: A coin is tossed 6 times. What is the probability that *all six tosses* will be tails?

Ex. 4: A die is rolled and a card is drawn from a standard 52-card deck. What is the probability that a red face card will be drawn and a number greater than 2 will be rolled?

Ex. 5: At an athletic event, athletes are tested for steroids using two different tests. The first test has a 93% probability of giving accurate results, while the second test is accurate 87% of the time. If both tests are used on a sample that does contain steroids, what is the probability that:

a) neither test shows that steroids are present?

b) both tests show that steroids are present?

c) at least one of the tests detects the steroids?

CW: p. 334 # 1-4, 15, 16

Dependent Events

A. Recall Independent Events

When the occurrence of one event, *A,* has *no effect* on the occurrence of another event, *B*, events *A* and *B* are said to be **independent**. The product rule for independent events states: **P(*A* and *B*) = P(*A*) x P(*B*)**

Ex. 1: Suppose you simultaneously roll a standard die and spin a spinner with eight equal sectors numbered 1 – 8. What is the probability of rolling an even number and spinning an odd number?

Rolling a die and spinning a spinner are *independent events* because the occurrence of one does not affect the occurrence of the other. We now turn our attention to cases where the probability of one event occurring *does affect* the outcome of another event.

B. Dependent Events

When the probable outcome of an event, *B*, depends directly on the outcome of another event, *A*, the events are said to be **dependent.** The **conditional probability** of *B*, written as P(*B* given *A*) or P($B|A$), is the probability that *B* will occur *given* that *A* has already occurred.

The product rule for two **dependent** events is the probability of the first event times the conditional probability of the second event given the first has occurred:

P(*A* and *B*) = P(*A*) x P(*B given A*) or **P(*A* and *B*) = P(*A*) x P($B|A$)**

Ex. 2: The probability that Mary will go to U of T is 0.2. The probability that she will go to another university is 0.5. If Mary goes to U of T, the probability that her partner Taylor will follow her and go to U of T is 0.75. What is the probability that both Mary and Taylor will attend U of T?

Ex. 3: In your pocket, you have a penny, a nickel and a dime. Determine the probability of reaching into your pocket and selecting a:

a) dime

b) dime then a nickel (without replacing the dime)

Ex. 4: The KCI Girls hockey team has eight wingers. Three of these wingers scored 4+ goals. If you choose two wingers at random, what is the probability of them both being 4+ goal scorers?

Ex. 5: A card is drawn from a standard deck of 52 cards. What is the probability that it is a jack, given that it is a face card?

CW: p. 334 # 5, 6; p. 358 # 9-11

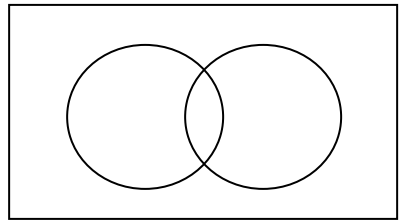
Mutually Exclusive and Non-Mutually Exclusive Events

A. Recall Venn Diagrams

When we looked at two events, *A* and *B,* that had some "overlap" (*intersection*) we noted that we could count the total number in A *or* B *or* Both (*union*) two ways:

1) *n*(A or B) = *n*(A *only*) + *n*(B *only*) + *n*(A *and* B)

2) *n*(A or B) = *n*(A total) + *n*(B total) – *n*(A *and* B)



These methods helped us avoid double-counting the number of outcomes in the event where A *and* B could occur together. We will use the same methods to help us avoid double-counting *probabilities* where two events could occur simultaneously!

B. Mutually Exclusive Events

Mutually exclusive events are events that **cannot occur simultaneously**. If one has occurred, it is not possible that the other occurred at the same time. Recall the tie example from *Unit 1*: since you cannot simultaneously *wear a tie* while *not wearing a tie*, these events are mutually exclusive. More examples of mutually exclusive events include:

- getting a 1 and a 2 on a single roll of a die
- getting a sum of 3 and two of a kind when rolling a pair of dice once
- removing a queen and an ace when taking one card from a standard deck

The addition rule for **mutually exclusive events** is (the probability of the first event) + (the probability of the second event), given there is no intersection of the events:

P(*A* or *B*) = P(*A*) + P(*B*)

Ex. 1: If two standard dice are rolled, what is the probability of rolling doubles *or* a total of 11?

Ex. 2: If a committee of five is to be randomly chosen from six males and eight females, what is the probability that the committee will be either all male or all female?

C. Non-Mutually Exclusive Events

Non-mutually exclusive events are events that **may occur simultaneously**. If one has occurred, it is quite possible that the other occurred at the same time. Recall the **Q♥** example from *Unit 1*: we were looking for the number of ways to select a queen *or* a heart from a standard deck of cards. Since you **could** select one or the other or both simultaneously, these events are *not mutually exclusive*. More examples of non-mutually exclusive events include:

- getting a 1 or a 2 on two rolls of a die
- getting a sum of 4 or two of a kind when rolling a pair of dice once
- removing a king or a spade when taking one card from a standard deck

The addition rule for **non-mutually exclusive events** is (the probability of the first event) + (the probability of the second event) – (the probability of both events):

P(*A* or *B*) = P(*A*) + P(*B*) – P(A *and* B)

Ex. 3: If two standard dice are rolled, what is the probability of rolling doubles *or* a total of 10?

Ex. 4: A card is randomly selected from a standard deck of cards. What is the probability that either a red card pr a face card will be selected?

Ex. 5: In Statsville, the probability of a teenager listening to the classic rock station is 42%, while the probability of listening to the New Country station is 36%. If 13% of the teenagers surveyed listen to both stations, what is the probability that a given student listens to neither of the stations?

Ex. 6: At Waterloo West Animal Hospital, where Mr. Jackson takes his puppy, the vet has found that Pipper will require his teeth cleaned with a probability of 0.6, his fur clipped with a probability of 0.3, and both with a probability of 0.1. (Hint: a Venn diagram is helpful . . .)

a) What is the probability that Pipper needs either his teeth cleaned or his fur clipped?
b) What is the probability that Pipper needs his teeth cleaned but not his fur clipped?
c) What is the probability that Pipper will require neither?

CW: p. 340-343 # 1 - 5, 11, 13, 15.

Probability Practise Worksheet II

A. Dependent and Independent Events

1. What is the probability that three standard dice rolled simultaneously will all land with the same number facing up?
2. Suppose you simultaneously roll a standard die and spin a spinner divided into 10 equal sectors, numbered 1 through 10. What is the probability of getting a 4 on both the die and the spinner?
3. If Aaron does his math homework today, the probability that he will do it tomorrow is 0.6. The probability that he will do it today is 0.4. What is the probability that he will do it both today and tomorrow?
4. What is the probability of rolling a total of 7 in two rolls of a standard die if you get an even number on the first roll??
5. A bag contains three green marbles and four black marbles. If Cristobal randomly picks two marbles from the bag at the same time, what is the probability that both marbles will be black?
6. If a satellite launch has a 97% chance of success, what is the probability of three consecutive successful launches?
7. A survey at a school asked students if they were ill with a cold or flu during the last month. The results were as follows. None of the students had both a cold and the flu.

	Cold	**Flu**	**Healthy**
Females	32	18	47
Males	25	19	38

Use these results to estimate the probability that:

a. A randomly selected student had a cold last month
b. A randomly selected female student was healthy last month
c. A randomly selected student who had the flu last month was male
d. A randomly selected male student had either a cold or the flu last month

Answers:

1) 1/36	2) 1/60	3) 0.24	4) 1/12	5) 2/7
6) 91.3%	7a) 57/179	7b) 47/97	7c) 19/37	7d) 22/41

B. Mutually Exclusive and Non-Mutually Exclusive Events

1. What is the probability of randomly selecting either a club *or* a non-face card from a standard deck of cards?

2. In a particular class, the probability of a student having blue eyes is 0.3, of having both blue eyes and blonde hair is 0.2, and of having neither blue eyes nor blonde hair is 0.5. What is the probability that a student in this class has blonde hair?

3. The probability that Jack will play golf today is 60%, the probability that he will play golf tomorrow is 75%, and the probability that he will play both days is 50%. What is the probability that he does *not* play golf on either day?

4. If 28% of the residents of Statsville wear contact lenses, 9% have blue eyes *and* wear contact lenses, and 44% have *neither* blue eyes *nor* wear contact lenses, what is the probability that a randomly selected resident has blue eyes?

5. If a survey of teenage readers of popular magazines shows that 38% subscribe to *Teen People*, 47% subscribe to *Cool Life*, and 35% subscribe to neither magazine, what is the probability that a randomly selected teenager:

    a. subscribes to both magazines?
    b. subscribes to either one magazine or both magazines?
    c. subscribes to only one of the magazines?

6. A survey of 50 female high-school athletes collected the following data:

Team	**Number of Athletes**
Field Hockey	23
Volleyball	16
Rugby	29
Both rugby and field hockey	8
Both rugby and volleyball	9
Both field hockey and volleyball	7
All three teams	6

    a. Draw a Venn diagram to illustrate the data.
    b. What is the probability that a randomly selected athlete will play on *only one* of the teams?
    c. What is the probability that a randomly selected rugby player also plays volleyball?
    d. What is the probability that a randomly selected athlete who does not play rugby is on the field hockey team?

Answers:

1) 43/52	2) 0.2			3) 15%	4) 37%
5a) 20%	5b) 65%	5c) 45%	6b) 19/25	6c) 9/29	6d) 15/21

MDM 4UI   Name: ______________________________

Sample TEST – Unit 3
Probability

INSTRUCTIONS
- ✓ Only neat, complete and organized solutions will receive full marks.
- ✓ Show all your work.
- ✓ All solutions must have accompanying event definitions, unless otherwise stated.
- ✓ Total Marks = 40

Useful formulas:

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ and } B) = P(A) \times P(B) \qquad P(A \text{ and } B) = P(A) \times P(B \mid A)$$

$$P(A \text{ or } B) = P(A) + P(B) \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Question 1 (2 marks)

Chris' sock drawer contains three pairs of grey socks, two pairs of white socks and four pairs of black socks. The socks are not matched or organized in any special way.

a) In a mad dash to get to school (and not walk into class late) Chris randomly selects a sock from his drawer without looking. What is the probability that he selects a grey sock?
b) Once this sock has been drawn, and discovered to be grey, what is the probability that Chris will select another grey sock to make a matching pair?

Question 2 (2 marks)

Keanan is immersed in an intense game of Yahtzee. Assuming the game is played with fair dice, find the probability of rolling:
a) a three or a five with one die.   b) two fives with two dice.

Question 3 (4 marks)

In Yu-Jin's bag of Smarties there are six red, five brown and three purple candies remaining. Suppose she selects two Smarties, one after the other, without replacing them. Find the probability that:
a) both Smarties are brown.
b) *at least* one Smartie is purple.

Question 4 (4 marks)

A six member work group is being created to plan details about KCI's new sports field. Five coaches and nine students are being considered for the committee. If the group is randomly selected, find the probability that it will include *at least* two coaches.

Question 5 (5 marks)

If a survey on teenage readers of popular magazines shows that 38% subscribe to *Teen People*, 47% subscribe to *Cool Life*, and 20% subscribe to both magazines, what is the probability that a randomly selected teenager subscribes to neither magazine?

Question 6 (4 marks)

Matt estimates that he has a 95% chance of passing Physics and a 99% chance of passing Data Management. Assuming that these are independent events,

a) Find the probability that Matt will pass both courses.
b) Find the probability that Matt will pass only one of the two courses.

Question 7 (4 marks)

A certain student determines that there is a 60% chance he will gather the courage to ask his sweetie to the prom. This student's friends assure him that if he asks, there is an 85% chance his sweetie will say yes. What is the probability our shy student will be seen at the prom with his sweetie?

Question 8 (4 marks)

A euchre deck contains 24 cards, the 9, 10, jack, queen, king and ace from each suit. If you were to deal out five cards from this deck, what is the probability that they will be a 10, jack, queen, king and ace all from the same suit?

Question 9 (5 marks)

When the KCI Senior Boys Football team has possession of the ball, the following empirical probabilities have been determined:

- The probability that the quarterback completes a pass is 0.6
- The probability that the quarterback completes a pass and they score a touchdown is 0.01

Find the probability that they score a touchdown given that the quarterback completes a pass.

Question 10 (6 marks)

The Blue Jays are underdogs in a best-of-five playoff series against Boston. The probability of the Jays winning each game is 0.375. If the Jays win the first game in this series, what is the probability they will win the series? (Hint: draw a tree diagram and consider each case that would lead to the Jays winning the series.)

Answers:

1a) 1/3	1b) 5/17	2a) 1/3	2b) 1/36	3a) 11%	3b) 40%	4) 76%
5) 35%	6a) 94%	6b) 5.9%	7) 51%	8) 0.009%	9) 1.66%	10) 48%

HW: p. 357 # 1 – 3, 6 – 8, 13 – 16; p. 360 # 1 – 5, 7 - 9