

# SPH3UI

## Unit 2: Kinematics

## Unit 2 Overview

Lesson	Text Reference	Topic	Questions	Done
1	1.1	Distance, Position and Displacement	Pg 11 P#1 – 3; Pg 13 P#1; Pg 13 Q#1 – 4, 6	
2	1.2	Speed & Velocity	Pg 20 Q#4, 6 – 8	
3	1.3	Acceleration	Pg 30 Q#4, 6 – 11; Activity: Hawk Diving	
Lab 2.1		Acceleration Due To Gravity	Lab Write-up & Discussion	
4	1.4	Comparing Graphs of Linear Motion	Pg 33 – Read Tutorial 1, P#1; Pg 35 Q#2 – 4 & Worksheet: Information in Kinematics Graphs	
5	1.5	Generating & Deriving Equations	Worksheet: Rearranging & Selecting Kinematics Equations	
6	1.5 & 1.6	Problem Solving	Pg 39 Q#2 – 4, 6; Pg 43 Q#4 – 7 Worksheet: Kinematics Problem Solving	
Lab 2.2		Motion of a Jeep	Lab Write-up & Discussion	
7	2.1	Vectors Using Scale Diagrams	Worksheet: Vector Problems (Scale) Pg 65 Q#7 – 9	
8	2.2	Vectors & Algebra	Worksheet: Vector Problems (Algebra) Pg 75 Q#1 – 5	
9	2.2	Relative Velocity	Pg 72 – Read Tutorial 4; Pg 74 P#2; Pg 75 Q#8, 9	
10	2.3	Projectile Motion	Worksheet: Projectile Problems Pg 81 Q#1 – 8	
Lab 2.3		Finding the Initial Velocity of a Projectile	Lab Write-up & Discussion	

# Lesson 1 – Distance, Position & Displacement

- ✓ All matter is in a constant state of *motion*.
- ✓ **Kinematics** is the study of how objects move.
- ✓ There are two types of motion: uniform and non-uniform.

**Uniform Motion:**

**Non-Uniform Motion:**

## Vector & Scalar Measurements

A scalar measurement has a \_\_\_\_\_ and a \_\_\_\_\_.

Examples:

A vector measurement has a \_\_\_\_\_, a \_\_\_\_\_ and a \_\_\_\_\_.

Examples:

### Notation

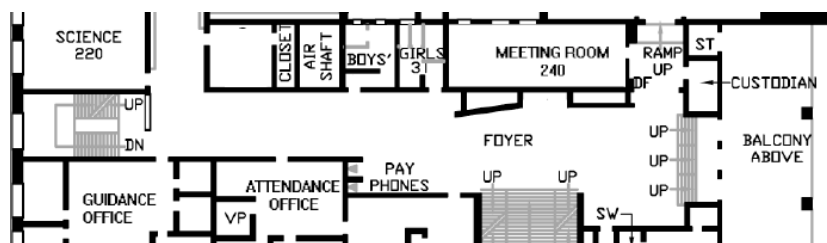
Vectors are distinguished from scalars by the addition of a special “hat” over top of any quantities symbol.

### Sign Conventions

Physicists will indicate the direction of motion using positive and negative signs. Unless stated otherwise, here is the normal assumption in all problems:

## Distance, Position and Displacement

Although similar, these three concepts are unique in our description of motion.



Distance

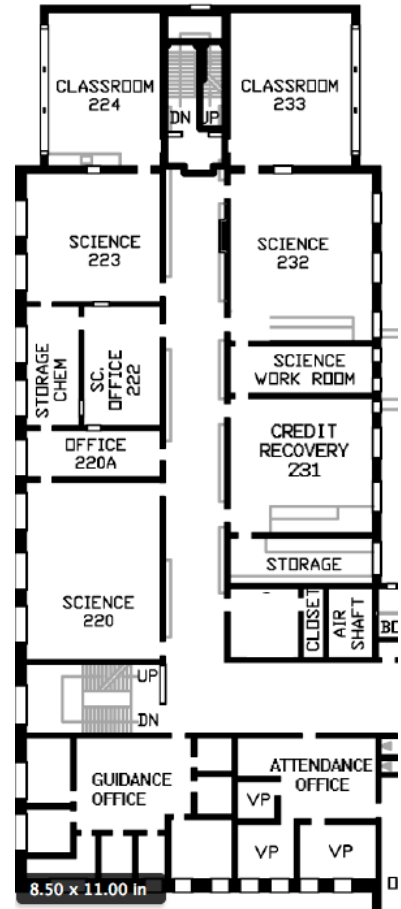
Position

Displacement

Example 1

Using the floor plan diagram shown, consider each of the following questions.

- You walk from the science office to room 220. What is your displacement?
- What is your displacement if you walk from room 231 to the guidance office?
- You walk from the science office, to the guidance office and then to VanB's room (232). What distance have you walked? What is your displacement?



Example 2

A dog is practicing for an agility competition. He leaves his trainer and runs 80 m due west to pick up a ball and then carries the ball 27 m due east and drops it into a bucket. What is the dog's total displacement?

## Lesson 2 – Speed and Velocity

- ✓ A position-time graph is a graphical description of motion.
- ✓ The **slope** of a position-time graph is the **velocity** of the object.
- ✓ Speed and velocity are similar but distinct characteristics in the study of motion.

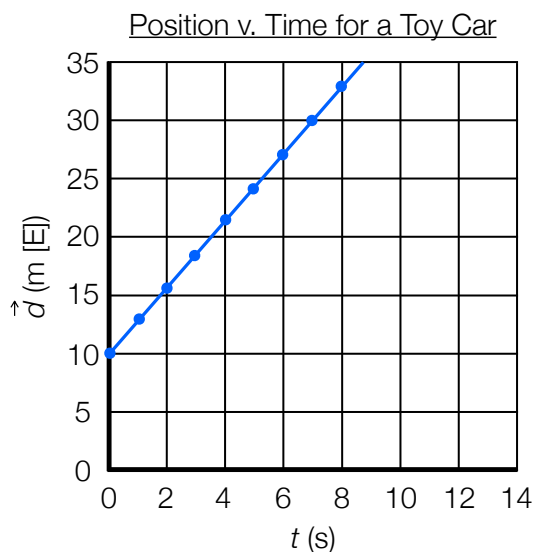
### Average Speed

For any object in motion, its average speed is its total distance travelled divided by the total time elapsed.



### Position – Time Graphs

A useful skill in the study of motion is the ability to extract information from the graph of an object's position versus time.



Reading the graph:

Finding the slope:



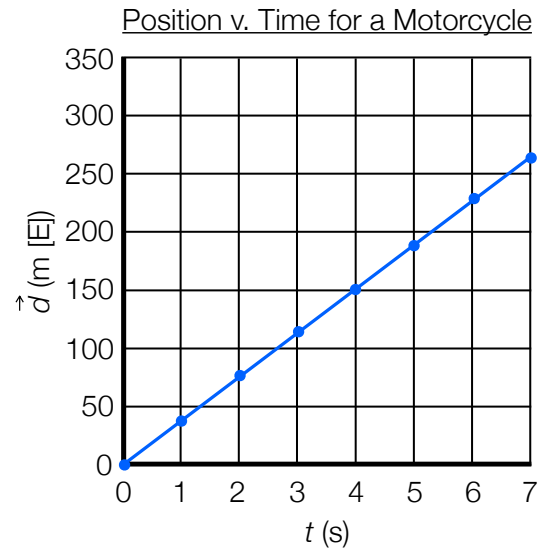
#### Example 1

A Lego train moves 1.25 m [E] and then 0.75 m [W] along its track in 8 seconds. Determine the average speed and the average velocity of the train.



Example 2

Consider the position-time graph shown. Determine the position of the motorcycle at 6 seconds and its average velocity during the time segment shown.



**Different Types of Motion; Different Graphs**

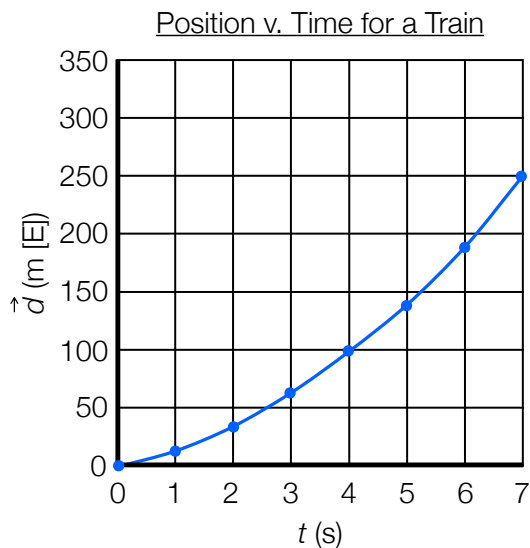
Position-time Graph	Description of Motion	Example
<p>A coordinate system with a vertical axis labeled <math>\vec{d}</math> (m [E]) and a horizontal axis labeled <math>t</math> (s). A horizontal blue line is drawn at a constant positive value on the y-axis.</p>	<ul style="list-style-type: none"> <li>✓ graph is a horizontal straight line</li> <li>✓ slope is zero; object has a velocity of zero; object is at rest</li> <li>✓ object is at a constant positive position (east) relative to a reference position</li> </ul>	
<p>A coordinate system with a vertical axis labeled <math>\vec{d}</math> (m [E]) and a horizontal axis labeled <math>t</math> (s). A horizontal blue line is drawn at a constant negative value on the y-axis.</p>	<ul style="list-style-type: none"> <li>✓ graph is a _____</li> <li>✓ slope is _____; object has velocity _____; object is _____</li> <li>✓ object is at a constant _____ position (____) relative to a reference position</li> </ul>	
<p>A coordinate system with a vertical axis labeled <math>\vec{d}</math> (m [E]) and a horizontal axis labeled <math>t</math> (s). A blue straight line starts at the origin and has a positive slope.</p>	<ul style="list-style-type: none"> <li>✓ graph is a straight line with positive slope</li> <li>✓ straight lines with non-zero slopes always represent <b>constant (non-zero) velocity</b></li> <li>✓ object is moving eastward, velocity can be found by determining slope</li> </ul>	
<p>A coordinate system with a vertical axis labeled <math>\vec{d}</math> (m [E]) and a horizontal axis labeled <math>t</math> (s). A blue straight line starts at a positive value on the y-axis and has a positive slope.</p>	<ul style="list-style-type: none"> <li>✓ graph is a _____</li> <li>✓ object has a _____ velocity</li> <li>✓ object is moving _____, velocity can be found by determining slope</li> <li>✓ object does not begin at the _____</li> </ul>	<p>A stick figure is shown running to the right. Below it is a horizontal arrow pointing to the right, indicating the direction of motion.</p>

## Lesson 3 – Acceleration

- ✓ Acceleration describes the change in velocity over time.
- ✓ The position-time graph for an accelerating object is a curve.
- ✓ The instantaneous velocity of an object is its velocity at that instant in time. It is equal to the slope of a tangent line to the position-time graph at that instant in time.
- ✓ The **slope** of a velocity-time graph is the **acceleration** of the object.
- ✓ The **area** under a velocity-time graph is the **displacement** of the object.

### Curved Position – Time Graphs

When an object is accelerating its position-time graph will not be a straight line, it will be a curve.

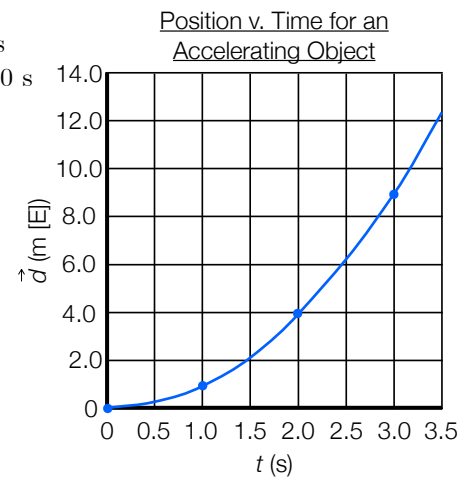


Reading the graph:

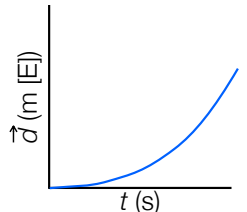
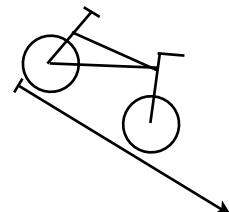
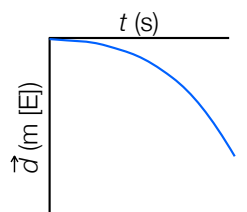
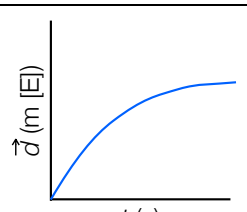
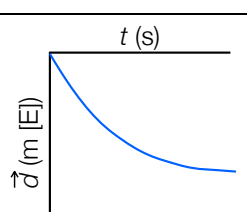
Finding the slope:

#### Example 1

Consider the position-time graph shown. Determine the instantaneous velocity of the object at 2.0 s and its average velocity over the first 2.0 s of motion.

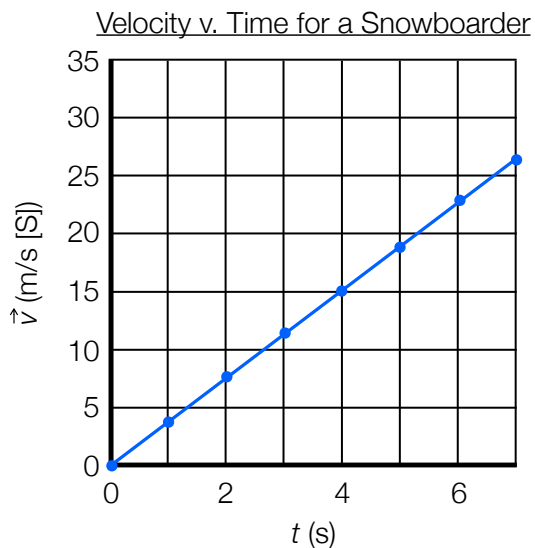


## Different Types of Motion; Different Graphs

Position-time Graph	Description of Motion	Example
 <p>The graph shows displacement <math>\vec{d}</math> (m [E]) on the vertical axis and time <math>t</math> (s) on the horizontal axis. A blue curve starts at the origin (0,0) and curves upwards with an increasing slope.</p>	<ul style="list-style-type: none"> <li>✓ graph is a curve</li> <li>✓ since the slope is changing, the velocity is not constant</li> <li>✓ The graph lies above the x-axis and its slope is increasing, the velocity of the object is increasing (speeding up) in a positive (eastward) direction</li> </ul>	
 <p>The graph shows displacement <math>\vec{d}</math> (m [E]) on the vertical axis and time <math>t</math> (s) on the horizontal axis. A blue curve starts at a positive displacement on the y-axis and curves downwards with a decreasing slope.</p>	<ul style="list-style-type: none"> <li>✓ graph is a curve</li> <li>✓ since the slope is changing, the velocity is not constant</li> <li>✓ The graph lies below the x-axis and its slope is _____, the velocity of the object is _____ (_____) in a _____ (_____) direction</li> </ul>	
 <p>The graph shows displacement <math>\vec{d}</math> (m [E]) on the vertical axis and time <math>t</math> (s) on the horizontal axis. A blue curve starts at the origin (0,0) and curves downwards with a decreasing slope.</p>	<ul style="list-style-type: none"> <li>✓ graph is a curve</li> <li>✓ since the slope is changing, the velocity is not constant</li> <li>✓ The graph lies above the x-axis and its slope is _____, the velocity of the object is _____ (_____) in a _____ (_____) direction</li> </ul>	
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## Velocity – Time Graphs

Acceleration, and other useful information, can be determined using a velocity-time graph.



Reading the graph:

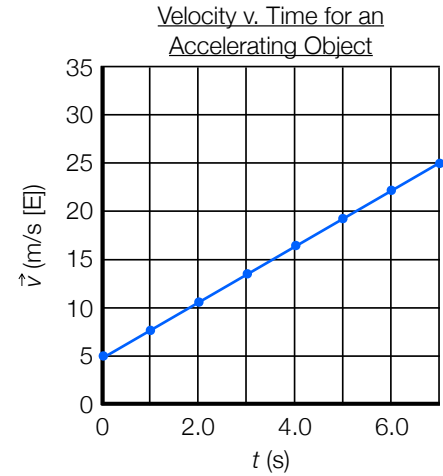
Finding the slope:

Area under the graph:



### Example 2

Consider the velocity-time graph shown. Determine the acceleration of the object and its displacement over the first 6.0 s of motion.



### Acceleration

Acceleration describes how quickly an object's velocity changes over time. This is seen as the slope of a velocity-time graph.

The units of acceleration are  $\text{m/s}^2$ . This is best thought of as:

### Example 3

A hockey player takes 1.3 s to accelerate from a slow skate to a full sprint. If the player's initial velocity is 2.65 m/s [N] and she can accelerate at  $1.77 \text{ m/s}^2$ , determine her final velocity at a full sprint.

## Activity

## Hawk Diving

There are many situations where motion is non-uniform, really anytime an object deviates from a straight line or changes speed. Because this type of motion is so common, we need to be able to analyse and make calculations from collected data.



The table below shows displacement versus time data for a hawk diving downwards at constant acceleration to catch a mouse.

<b>Time (s)</b>	0.0	1.0	2.0	3.0	4.0	5.0	6.0
<b>Displacement (m) [down]</b>	0.0	2.0	8.0	18.0	32.0	50.0	72.0

### Instructions

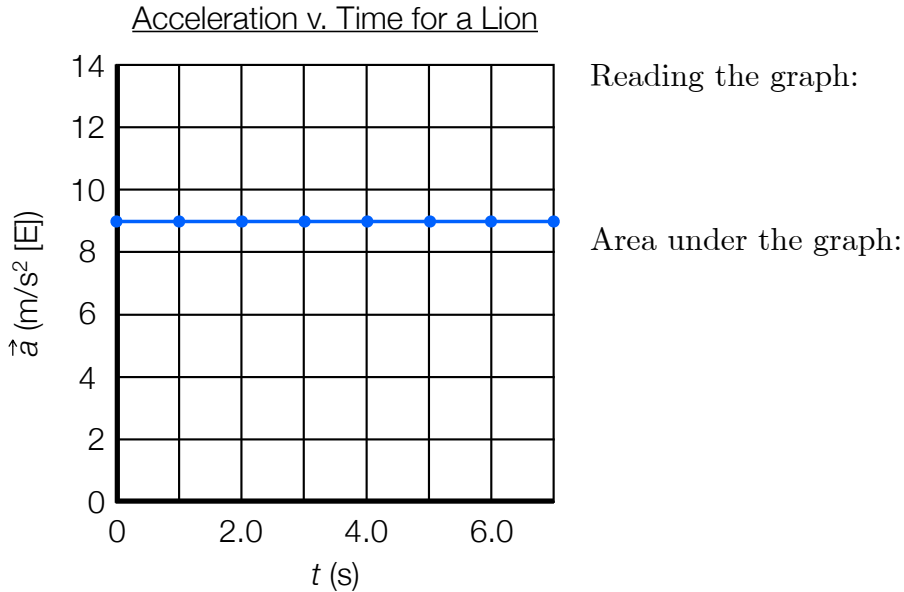
- 1) Complete all work on graph paper.
- 2) Draw a proper displacement-time graph of the data. Be accurate. Place time on the horizontal axis and displacement on the vertical axis. Connect the dots using a “French Curve” – a tool designed to draw curved lines.
- 3) You know that we cannot find the slope of a curved line. Find the instantaneous velocity of the hawk, using tangents to the curve, at 1.0, 2.0, 3.0, 4.0 and 5.0 s.
- 4) On a second piece of graph paper, place the velocities you just found onto a velocity-time graph for the hawk. Place time on the horizontal axis and velocity on the vertical axis. What do you notice about the points on this graph? Connect them using a straight line of best fit.
- 5) Calculate the slope of this line. Consider the units of the slope. What information does the slope of a line on a  $\vec{v}-t$  graph tell us about the motion?

# Lesson 4 – Comparing Graphs of Linear Motion

- ✓ The **area** under an acceleration-time graph is the **velocity** of the object.
- ✓ Given any of the three motion graphs, it is possible to construct the others using data found in the original.

## Acceleration – Time Graphs

When dealing with uniform (or constant) acceleration, these graphs will always be a horizontal line. In high school kinematics, we only encounter situations involving uniform acceleration.

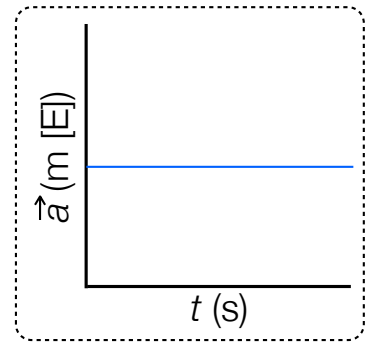
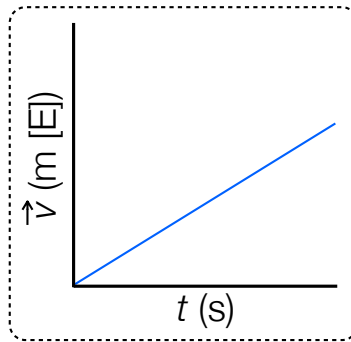
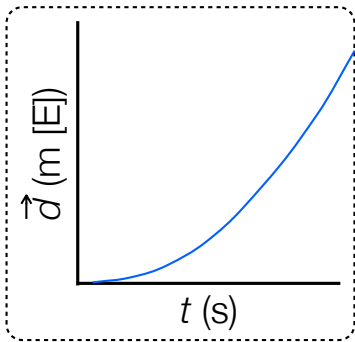


## Examining Relationships Between Graphs

Velocity-time Graph	Acceleration-time Graph	Motion	Example

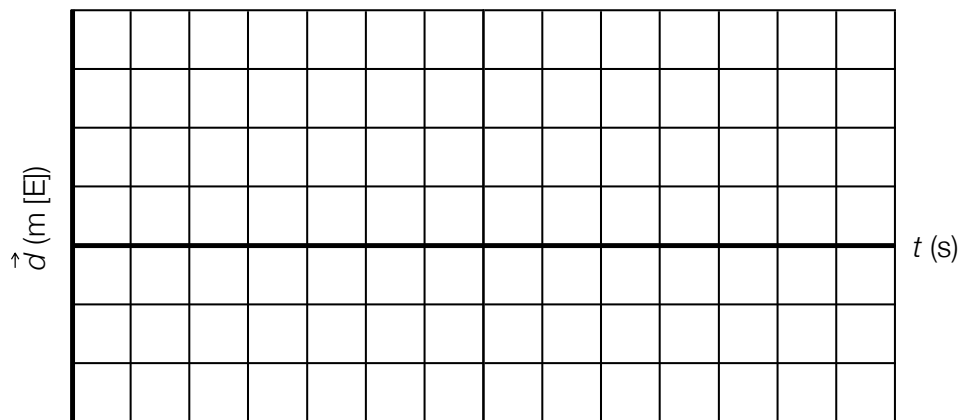
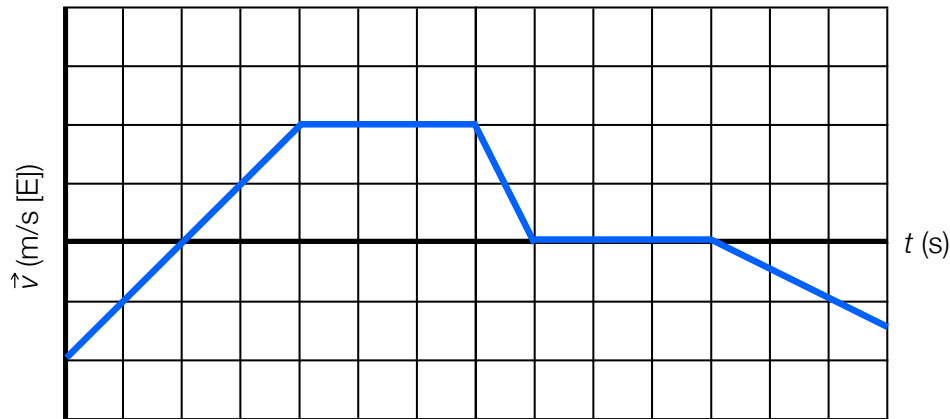
## Graphical Analysis with Linear Motion Graphs

It is important to be able to analyze and interpret the information found in different motion graphs. Information can be found by reading the graph, finding the slope, or determining the area of the graph. The graphic below illustrates the options available.



### Example 1

Using the velocity-time graph shown below, construct the corresponding displacement-time graph.

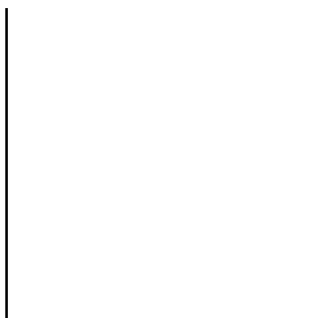


- ✓ There are five **key equations** that describe motion involving uniform acceleration.
- ✓ These equations can be derived from a general velocity-time graph and simple algebra.

When it comes to solving problems equations are extremely useful. Algebraic methods are usually quicker and more convenient than graphing. There are 5 standard kinematics equations and this lesson is dedicated to their derivation from the standard kinematics graphs.

### **Equations for Displacement and Acceleration**

In our previous lessons, we have done most of the work to find the first two equations. Recall that we can do a lot with a general velocity-time graph:



To generate the first equation, find the slope:

And rearrange to obtain a linear equation,

To generate the second equation, we determine the displacement from the area under the curve:

### **Additional Motion Equations**

The 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> equations come from these first two, but are expressed in different ways (with different variables). We use the method of substitution to solve for the final three equations.

Substitute the expression for  $\vec{v}_2$  in Equation 1 into Equation 2:

Similarly, rearrange Equation 1 for  $\vec{v}_1$  and then substitute this expression into Equation 2:

Finally, isolate  $\Delta t$  in Equation 1 and substitute into Equation 2:

And there you have it! These are the five fundamental equations used in kinematics. Note that they are derived from graphs containing constant acceleration; *they are only applicable in problems that have uniform (constant) acceleration.*

Knowing the equations and where they come from is the first part of the challenge, learning how to use the equations to solve problems in the second part. Over the next few days we will use these equations to solve tons of different kinematics problems.

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \quad \textcircled{1}$$

$$\Delta \vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t \quad \textcircled{2}$$

$$\Delta \vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2 \quad \textcircled{3}$$

$$\Delta \vec{d} = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}\Delta t^2 \quad \textcircled{4}$$

$$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta \vec{d} \quad \textcircled{5}$$

## Lesson 6 – Problem Solving

- ✓ A method (such as G.R.A.S.S.) can help make problem solving more efficient.
- ✓ The symbol  $g$  is used to represent the acceleration due to gravity.
- ✓ All objects in free fall near to Earth's surface will accelerate at  $9.8 \text{ m/s}^2$  in a direction toward the centre of the Earth.

We have already solved a few simple problems as they relate to displacement, velocity and acceleration. But with the addition of vectors, new equations and variables, questions can be a little bit trickier.

### Example 1

A skate boarder goes down a hill at  $4.0 \text{ m/s}$  and accelerates at  $2.0 \text{ m/s}^2$ . What is their velocity  $5.0 \text{ s}$  later?

### Example 2

A Tundra pick-up truck travels at  $10 \text{ m/s [N]}$ . It accelerates at  $4.0 \text{ m/s}^2 \text{ [N]}$  for  $8.0 \text{ s}$ . What is the truck's displacement?

### Example 3

A ball rolls up a hill at  $4.0 \text{ m/s}$ .  $5.0 \text{ s}$  later, it rolls down the hill at  $6.0 \text{ m/s}$ . What is its displacement after  $5.0 \text{ s}$ ?

Example 4

A boy spits a watermelon seed down from a 40.0 m balcony at 2.0 m/s. Assuming no air resistance, how fast is the seed going after 3.0 s? Acceleration due to gravity is  $9.8 \text{ m/s}^2$ .

Example 5

A physics student drops a penny (from rest) off of the roof of KCI. If there is no resistance from the air, how far has the penny traveled after 2.5 s?

Example 6

A motorcycle travels at 45 m/s [W] through a school zone. How fast will it be going if the driver hits the brakes and decelerates at  $5 \text{ m/s}^2$  for 3.0 s?



# Worksheet

## Kinematics Problem Solving

Solve the following problems showing necessary steps. Remember that when you are working with variables in two opposite directions; be sure to assign one of the directions as positive and the other direction as negative. Also, if gravity is involved, remember that the acceleration due to gravity is equal to  $9.80 \text{ m/s}^2$ .

- Graham threw a volleyball straight up with an initial speed of  $21.0 \text{ m/s}$ .
  - How high above the release point did it go?
  - How long did it take to reach its maximum height?
  - How long did it take to return to the release point?
- Mackenzie was leaning over the edge of a cliff  $32.0 \text{ m}$  high. She threw a stone straight up at a speed of  $12.0 \text{ m/s}$ . How long did it take for the stone to hit the ground?
- A train is  $120 \text{ m}$  long. Sydney was standing  $200 \text{ m}$  from the front of the train when it began to accelerate from rest. She noticed that the front of the train was moving at  $8.00 \text{ m/s}$  when it passed her. How fast would the back of the train be going when it passed her?
- Sofia was driving at  $90.0 \text{ km/h}$  when she saw the light turn yellow at an intersection located  $65.0 \text{ m}$  ahead. She used  $0.40 \text{ s}$  to decide what to do, and then braked at  $-5.0 \text{ m/s}^2$  until she stopped. What was her stopping distance? Did she stop before she reached the intersection?
- The barrel of a rifle is  $72.0 \text{ cm}$  long. If it can fire a bullet at a speed of  $360 \text{ m/s}$ , what would be the acceleration of the bullet?
- Kelton kicked a rugby ball from ground level straight up at a speed of  $22.0 \text{ m/s}$ . How long would it take for the ball to reach a height of  $16.0 \text{ m}$ ?
- Adam threw a baseball straight up at a speed of  $24.0 \text{ m/s}$ . How long would it take for the ball to reach a height of  $33.0 \text{ m}$ ?

- a)  $22.5 \text{ m}$  b)  $2.1 \text{ s}$  c)  $4.3 \text{ s}$
- $4.06 \text{ s}$
- $10.1 \text{ m/s}$
- $72.5 \text{ m}$
- $9.0 \times 10^4 \text{ m/s}^2$
- $3.58 \text{ s}$  and  $0.91 \text{ s}$
- The ball does not reach this height.

Answers:

## Lesson 7 – Vectors Using Scale Diagrams

- ✓ Objects can move in two dimensions, such as a horizontal plane and a vertical plane.
- ✓ The compass can be used to express directions in the horizontal plane.
- ✓ Displacement vectors can be added using a scale diagram.

Vectors are different than scalar values because they include:

Vectors are represented graphically as a directed line segment, that is to say, an arrow pointing in the given direction.

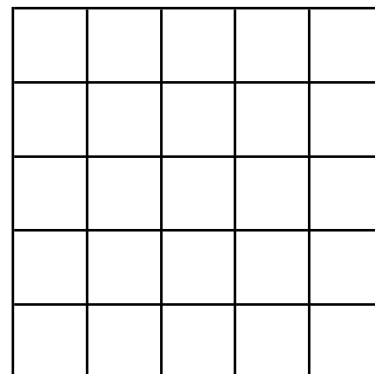
To use vectors in problems we need to be able to manipulate them in a similar way to scalars. We will **ONLY** concern ourselves with vector addition and subtraction for the purposes of this course.

In addition to this, vectors can be manipulated primarily in two different ways: graphically and algebraically. We will begin with the graphical manipulation of vectors and then examine the algebraic manipulation of vectors next lesson.

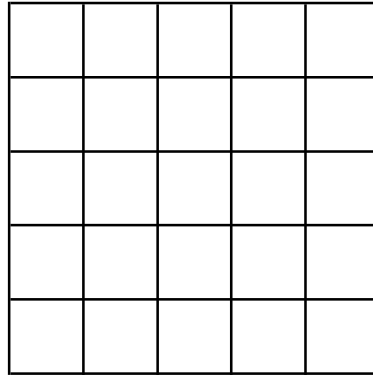
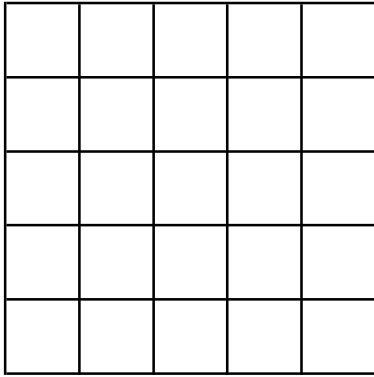
### **Vector Problems**

#### Example 1

Complete the following vector problems using your protractor and ruler. Answer each problem by stating the resultant vector. All problems should be answered to scale.

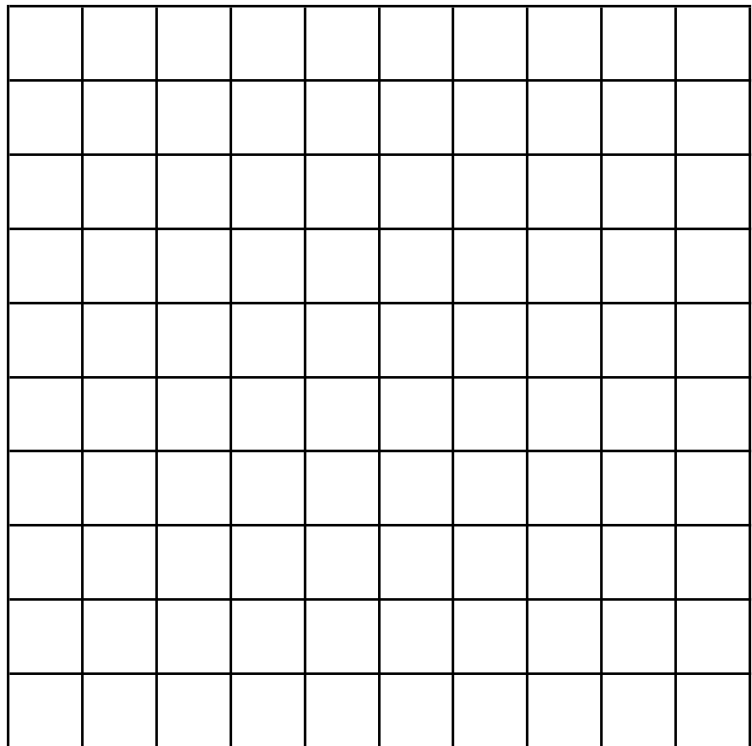


Example 1 (con't)



Example 2

A student walks 7.5 m [N] and then 6.0 m [W]. Draw a vector diagram and indicate the student's final displacement.



**Worksheet**

**Vector Problems (Scale Diagrams)**

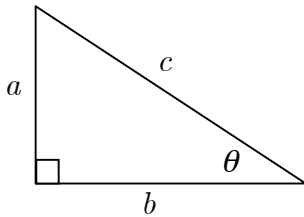
For each of the following, draw a vector diagram and find the final displacement. Diagrams must be to scale.

- 1) A dog runs 2.4 km [W] and then 4.6 km [S]
- 2) A flea jumps 7.0 mm [N] and then 2.5 mm [W]
- 3) A car drives 80 km [N] and then 40 km [S]
- 4) A leaf blows 4.5 m [W] and then 4.5 m [E]

- ✓ Perpendicular vectors can be added algebraically using trigonometry and Pythagorus.
- ✓ The component method of vector addition will allow us to add any and all vectors.

Often it is more convenient (and less time consuming) to complete vector problems using an algebraic method, simply because we don't need to worry about completing scale diagrams if we use algebra.

Recall that for any right-angle triangle, the following hold:



Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Trigonometric Ratios

$$\sin \theta = \frac{a}{c} \quad \cos \theta = \frac{b}{c} \quad \tan \theta = \frac{a}{b}$$

We will use the above information to solve vector problems from now on. The completion of these types of questions can be a tedious task. If you observe the following general steps, you'll be completing complex vector problems in no time.

Step 1

Step 2

Step 3

Step 4

Example 1

Break the vector 5.0 cm [N 65° E] into two perpendicular component vectors.

### Example 2

A prowling lion moves 12.0 m [W 30° S] and then 4.0 m [N]. Find the lion's displacement.

## Worksheet

## Vector Problems (Algebra)

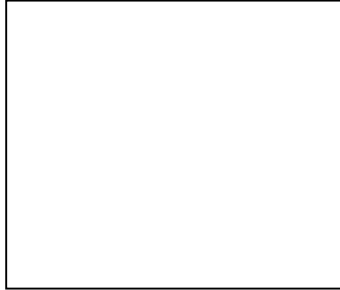
Draw a *sketch* and then use the algebraic method of vector addition to find the displacement for the following.

- A skittish squirrel darts 12.0 m [W] and then 7.0 m [S].
- A nervous chicken takes the following route to avoid a KFC: 4.0 km [S], 1.0 km [E], 1.0 km [N], 1.0 km [W], 1.0 km [S], 5.0 km [W].
- A boat sails 4.5 km [S], then 6.0 km [NW] and finally 9.0 km [E 15° N].
- Two boy scouts are competing in a team orienteering event. They follow instructions to go 50 m [N], 75 m [N 45° E] and then 100 m [S 30° E].
- Alphonse and Beauregard enter into a car rally competition and travel 4.0 km [SE], 3.5 km [E], 5.0 km [N] and 4.5 km [NW].
- A bird flies 15 km [S 25° E], 20 km [W 30° S], 45 km [E 5° N] and then 25 km [N 20° E].

- ✓ We can use vector addition to add velocity vectors as well as displacement vectors.
- ✓ River-crossing and air-navigation problems are examples where we use relative velocity.

### Relative Motion in One Dimension

Consider the three experiments below. We will use these in order to begin to describe the idea of relative velocity.



In each of the cases above we have measured a different velocity depending on the situation. What is common in each case is that the velocity is always measured **relative** to a **reference frame**. Most of the time, in everyday life, we use the ground as a standard reference frame.

In order to distinguish between the different velocities involved in what we are doing, we often use a series of subscripts:

$$\vec{v}_{\text{og}}$$

$$\vec{v}_{\text{mg}}$$

$$\vec{v}_{\text{om}}$$

The relationship between the three quantities is

$$\vec{v}_{\text{og}} = \vec{v}_{\text{om}} + \vec{v}_{\text{mg}}$$

#### Example 1

A student swims in a river. When the current flows, its velocity is 1.0 m/s [E]. The student can swim at 2.0 m/s (in still water). Find the swimmer's velocity with respect to the ground when:

- a) there is no current flowing
- b) The current is flowing and the swimmer swims with the current
- c) The current is flowing and the swimmer swims against the current.

## Relative Motion in Two Dimensions

When dealing with relative motion and velocities in two dimensions, add the vectors in a similar manner to the previous example. The method is very similar to adding vector displacements. The difficult part is to determine what each velocity is measured with respect to.

### Example 2

If a student can swim at 2.0 m/s with respect to still water and there is a current of 1.0 m/s [E], find the student's speed with respect to the ground if he is seen to swim straight across the river.

### Example 3

A pilot with a heading of [N30°E] and an airspeed of 400 km/h flies into a wind coming from the north at 110 km/h. What is the plane's velocity with respect to the ground?

- ✓ A projectile is any object that moves along a two-dimensional trajectory under the influence of gravity only.
- ✓ Projectile motion consists of independent horizontal and vertical motions. The horizontal and vertical motions take the same amount of time.
- ✓ Objects can be projected horizontally or at an angle to the horizontal. Projectile motion can begin and end at the same or different heights.
- ✓ The five kinematics equations can be used to solve projectile motion problems.

## Parabolic Motion

When you throw a ball in outer space, the ball will leave your hand and travel in a straight line forever. (Unless it runs into an object or encounters a gravitational field.) On Earth, this does not happen. When we throw a ball on Earth, it travels not in a straight line but in an arc.

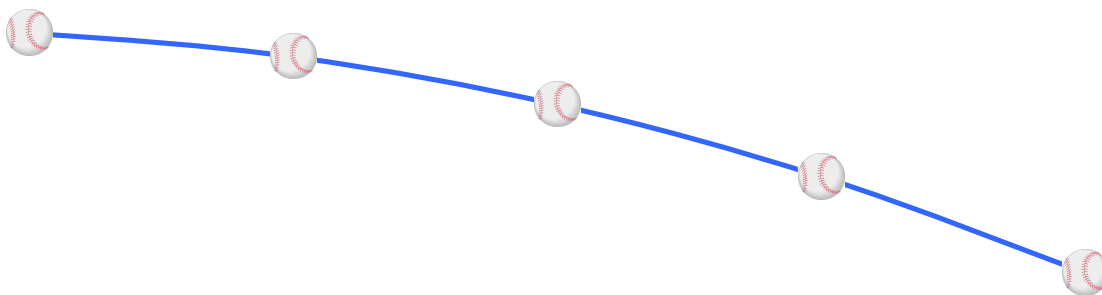


The term **projectile motion** is used to describe events where an object travels along a two-dimensional path and moves under the influence of gravity (it is not self powered).

If we analyze the motion more specifically, we make the following observations:

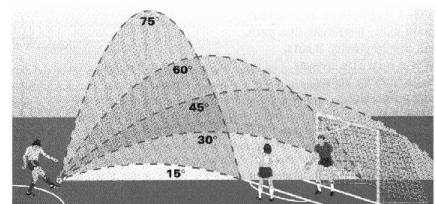


And the trajectory of the ball is shown below:



Notice that each horizontal component of the velocity is *constant* but that each vertical component of velocity *increases*. This is because of the acceleration due to gravity.

The angle of launch also affects the path of a projectile, specifically the **range** which is the displacement in the horizontal direction,  $x$ .





Example 1

An arrow is shot from a height of 20.0 m above the ground with an initial horizontal velocity of 18.0 m/s. How long will it take the arrow to reach the ground? How far will the arrow travel in the horizontal direction?

The key to solving these two-dimensional problems is to break them up into two separate one-dimensional parts, just like in the preceding example. In your final answer you can then recombine the parts to give a two-dimensional solution. This means that you will handle  $x$  and  $y$  components separately, each with their own set of givens, unknowns and equations.

Example 2

A soccer player on a level playing field kicks a ball with a velocity of 9.4 m/s at an angle of  $40^\circ$  above the horizontal. Determine the soccer ball's range and maximum height.

### Example 3

A golf ball is hit with an initial velocity of 30 m/s, at an incline of  $50^\circ$ . It lands 2.0 m above the point of impact and 70 m away. Find the final velocity of the golf ball, just before it hits the ground.

## Worksheet

### Projectile Motion Questions

1. A white-tailed deer can jump a fence with a *maximum height* of 2.4 m. If the deer jumps up at an angle of  $45^\circ$ , what must be the deer's initial speed?
2. Nadia threw a rubber ball from a height of 1.24 m at a speed of 15.0 m/s and at an angle of  $22^\circ$  **below** the horizon.
  - a. How far away from Nadia (in the horizontal direction) did the ball hit the ground?
  - b. How fast is the ball going when it hits the ground?
  - c. At what angle to the horizon does it hit the ground?
3. Beaupre can throw a rugby ball with a speed of 28.0 m/s. Calculate the theoretical maximum range of the rugby ball.

### Answers:

1. 9.7 m/s      2a) 2.64 m      2b) 15.8 m/s      2c)  $28^\circ$  below      2d) 2.30 m      2e) 17.2 m      3. 80.0 m